Fermilab

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Linear Optics Measurements

Contents:

- Optics measurements:
  - LOCO
  - TBT analysis: Fourier analysis, ICA
  - Tevatron Linear coupling correction

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Focusing errors and Optics Measurements

Errors in the focusing elements lead to an optics perturbation. In first approximation[a]

$$\frac{d^2}{d\phi_z^2}\left(\frac{\Delta\beta_z}{\beta_z}\right) + 4Q_z^2\left(\frac{\Delta\beta_z}{\beta_z}\right) = -2Q_z^2\beta_z^2\Delta K \qquad \phi_z \equiv \mu_z/Q_z$$

Closed solution for an integrated gradient error, $\Delta K\ell$, at $s = s_k$

$$\frac{\Delta\beta_z}{\beta_z}(s) = -\frac{1}{2\sin(2\pi Q_z)}\beta_z^k \cos[2Q_z\pi - 2|\mu_z(s) - \mu_z^k|]\Delta K\ell$$

Beta-beating

- oscillates with *twice* the betatron frequency and is thus sensitive to error harmonics near to $2Q_z$
- is large when Q_z approaches a *half integer*

[a] see Courant-Snyder

Errors in the focusing structure have bad consequences

- *unpredictable response* to any machine parameter change
- *uncontrolled beam size* with consequences on aperture, luminosity, beams separation (Tevatron).

The β function value at a quadrupole location may be evaluated by changing its current and measuring the tune change

$$\beta_z = -4\pi \frac{\Delta Q_z}{\Delta K \ell}$$

This is a good old method, but

- it requires *independently* powered quadrupoles
- results are affected by
  - orbit perturbations arising from the beam being *off-center* at the quadrupole
  - magnet *histeresys* (Debuncher, Accumulator)
- the quadrupole *calibration* must be well known

Today BPM systems allow more sophisticated techniques and several methods for measuring the linear optics and fitting measurement to a model have been developed in the last years.

Two main *philosophies*:

- Closed Orbit response to the excitation of correctors
- Analisys of beam oscillations excited by single kicks or AC dipoles (TBT analysis); data acquisition is fast and, unlike the previous method, it may be applied in fast cycling machines (Booster)

ORM

Orbit change due to the a corrector (no coupling)

$$\delta z_i = T_{ij}\Theta_j = \frac{1}{2\sin\left(\pi Q_z\right)}\sqrt{\beta^m_{z,i}\beta^c_{z,j}}\cos\left(Q_z\pi - |\mu^m_{z,i} - \mu^c_{z,j}|\right)$$

with $z = x$ or y. The response is proportional to β values both at corrector and monitor position as well it depends on the phase advance between them. In presence of errors the *actual* Twiss parameters may be determined by *measuring* the actual orbit response matrix.

By powering one corrector and reading its effect on *all* BPMs one get NBPM conditions and 2×NBPM + 2 unknowns. By using *all* correctors the number of unknown parameters increases to 2×NBPM + 2×NCOR but the number of constraints becomes NBPM×NCOR. The number of unknown parameters increases by 2×NBPM + 2×NCOR if also BPM and correctors roll angles and calibrations are considered.

The (usually) large number of constraints allows to compute accurately the unknown parameters at BPMs and correctors by simple computations [a].

One can do more by attempting to change the *machine theoretical model* so to fit the measured orbit changes. In general (eventually coupled machine)

$$\vec{Z} = M^{meas}\vec{\Theta} \qquad \text{with } \vec{Z} \equiv \begin{pmatrix} x \\ y \end{pmatrix} \qquad \vec{\Theta} \equiv \begin{pmatrix} \Theta_x \\ \Theta_y \end{pmatrix}$$

M_{ij}^{meas} being the measured beam position at the i^{th} BPM due to a unitary kick at the j^{th} corrector. One can *compute* the response matrix, M^{mod}, for the theoretical optics, by using any (coupled motion handling) optics code. Machine parameters as quadrupole gradients, roll angles etc., as well as gauge factors and roll angles of BPMs and correctors are varied so to minimize the difference between the model matrix and the measured one

$$\chi^2 = \Sigma_{ij}\frac{[M_{ij}^{mod} - M_{ij}^{meas}]^2}{\sigma_i^2} \qquad \sigma_i \equiv \text{BPMs rms noise}$$

[a] the equations being non-linear in the unknown parameters, one may apply an iterative procedure

Two ways for solving in practice the problem

- CALIF orginal algorithm by Corbett, Lee and Ziemann (SLAC) uses a first-order perturbation

$$M_{ij}^{mod} = M_{ij}^{mod,0} + \Sigma_q \frac{\partial M_{ij}^{mod,0}}{\partial K_q} \Delta K_q$$

  with $M_{ij}^{mod,0}$ and its derivatives computed by COMFORT. Gauge errors are embedded into M_{ij}^{meas}

- LOCO (Linear Optics from Closed Orbits) by Safranek (BNL) iterates the above procedure recomputing $M_{ij}^{mod,0}$ and derivatives at each step. It is slower but more accurate.

These techniques were first developed for *small* machines as SPEAR (SLAC) and the NSLS X-Ray Ring (BNL). At Fermilab LOCO has been introduced with support from Sajaev of ANL, by Lebedev, Nagaslaev, Valishev and M. Xiao for Recycler, Debuncher, Antiproton Accumulator and Tevatron.

Tevatron is the *largest* accelerator where such technique is routinely used. The upgrade of the BPM system improved the position measurement resolution to about 10μm. Ideally the lower limit to the difference between model-expected orbit and measured orbit is the BPMs resolution.

- 29+30 out of 2×110 correctors tipically used
- 118 horizontal and 118 vertical BPMs
- 216 normal and 216 skew quadrupole errors

The model includes also the (unknown) beam energy change due to the *horizontal* correctors

$$M_{ij}^{model} \longrightarrow M_{ij}^{model} + \frac{\Delta E_j}{E} D_{x,i}^{m} \qquad \text{with} \qquad \frac{\Delta E_j}{E}\Theta_j = -\frac{D_{x,j}^{c}}{\alpha_c \mathcal{L}}\Theta_j$$

$$\Rightarrow 1022 + 29 \text{ free parameters and } 13924 \text{ equations.}$$

The dispersion at the BPMs is measured as usual by changing the RF frequency.

Wrt the original ANL version, the energy change effect and coupling where introduced. Both were essential to get *sensible* fits. Nevertheless a lot of "manual" tweaking was necessary before the code produced reasonable results.

Using 59 correctors and recording 20 orbits per corrector setting, data acquisition requires about 50 minutes.

Data analysis is *parallelized* and runs on the linux HEIMDALL cluster. The model is computed by OptiM, starting with *actual* machine currents and magnet survey data. The fit takes about 2 hours (assuming model and measurement are not too far).

A mathematical "excursus":

Singular Value Decomposition

Any $\boldsymbol{M} \times \boldsymbol{N}$ (with $\boldsymbol{M} \geq \boldsymbol{N}$) matrix $\boldsymbol{A}$ may be written as

$$\boldsymbol{A} = \boldsymbol{U \Sigma V^T}$$

with

$\boldsymbol{U}$	$\boldsymbol{M} \times \boldsymbol{N}$ orthogonal matrix ie $(\boldsymbol{U^T U})_{\boldsymbol{ij}} = \boldsymbol{\delta_{ij}} \;\; \boldsymbol{i, j} \leq \boldsymbol{N}$
$\boldsymbol{V}$	$\boldsymbol{N} \times \boldsymbol{N}$ orthogonal matrix
$\boldsymbol{\Sigma}$	$\boldsymbol{N} \times \boldsymbol{N}$ diagonal matrix

The diagonal elements $\boldsymbol{\Sigma_j}$ are called *singular values*.

For $\boldsymbol{M} < \boldsymbol{N}$, it is $\boldsymbol{\Sigma_j} = \boldsymbol{0}$ and $\boldsymbol{U_{ij}} = \boldsymbol{0}$ for $\boldsymbol{j} > \boldsymbol{M}$.

There are routines available for matrix decomposition.

SVD decomposition is useful for solving system of linear equations.

For a squared matrix, the inverse is

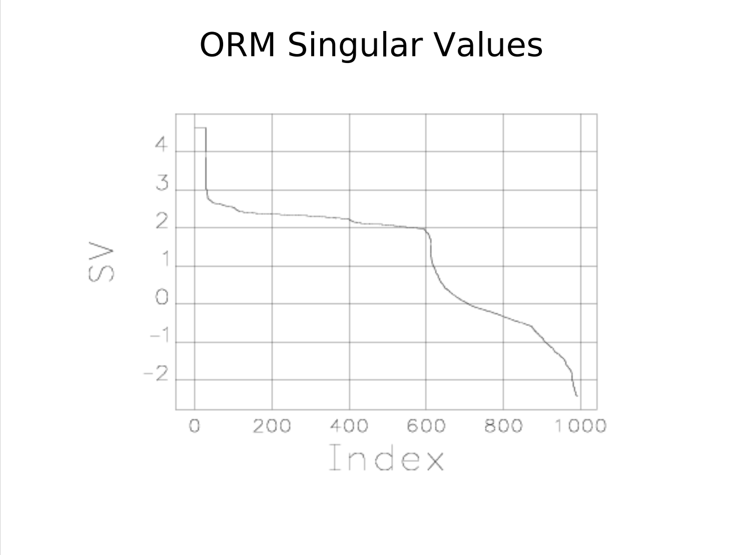
$$\boldsymbol{A}^{-1} = \boldsymbol{V}\boldsymbol{\Sigma}^{-1}\boldsymbol{U}^{T}$$

If some $\boldsymbol{\Sigma}_i = \mathbf{0}$ (or small) the matrix cannot be inverted and the system has infinite number of solutions. Replacing $\mathbf{1}/\boldsymbol{\Sigma}_i$ by zero for those singular values, the matrix $\boldsymbol{V}\boldsymbol{\Sigma}^{-1}\boldsymbol{U}^{T}$ gives *the* solution with the *smallest* length.

For $\boldsymbol{M} < \boldsymbol{N}$ (more variables than constraints), again the decomposition will give the smallest length solution, while for the case $\boldsymbol{M} > \boldsymbol{N}$ (more constraints than variables) it gives the *least square* solution.

Excursus end.

The LOCO mathematical solution at each step is found by a SVD with threshold adjusted to eliminate the very small singular values.



Problems (common to other methods)

- it can only resolve relative BPMs and corrector gauge errors
- large kicks increase signal-to-noise ratio, but introduce large systematic errors (non-linearities)

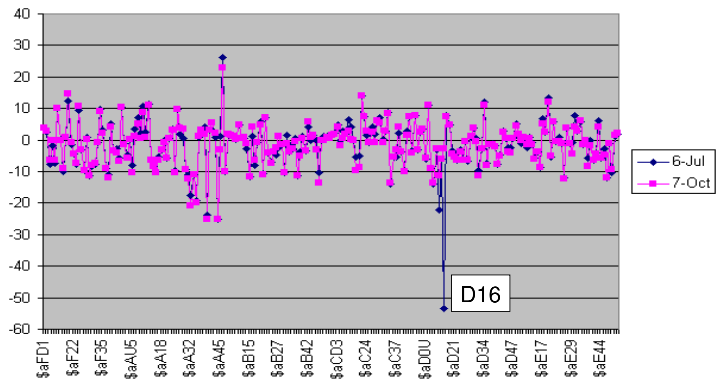
The effect of random errors on the fitted solution is estimated by comparing the solution found for a single set of measurement with the average value.

The LOCO analysis applied to the Debuncher allowed to improve the optics and to increase the machine aperture.

The LOCO analysis is used routinely at Tevatron to

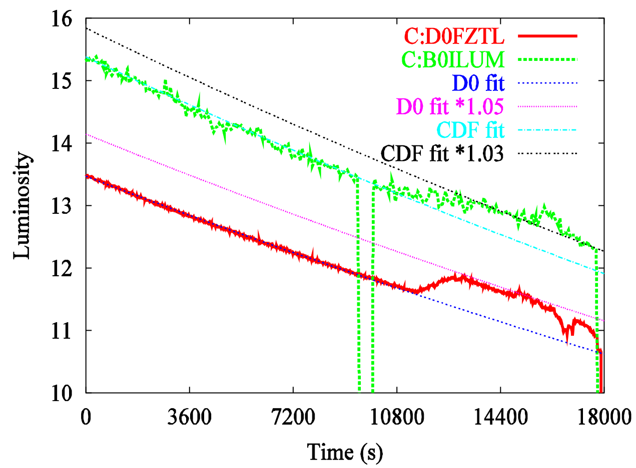
- find *large* gradient/roll errors
- keep track of the machine optics.
- produce a *realistic model* as basis for optics improvement and luminosity gain

Skew Gradient Errors (kGs)

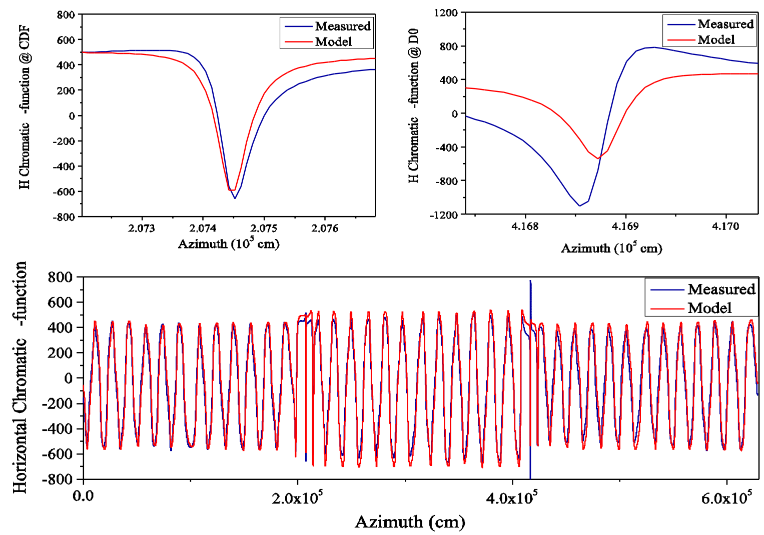


(A. Valishev courtesy)

28cm β* End of Store Study 8/4



Horizontal Chromatic Beta Function



Fourier Analysis of TBT data

Single Kick

The Fourier analysis of TBT data has been first applied at LEP in 1992 as a tool for measuring the *uncoupled linear optics*.

TBT data at the j[th] BPM following a single kick in the z plane ($z \equiv x, y$)

$$z_n^j = \frac{1}{2}\sqrt{\beta_z^j}\mathrm{e}^{i\Phi_z^j} A_z \mathrm{e}^{iQ_z(\theta_j+2\pi n)} + c.c.$$

$$\text{with} \quad n \equiv \text{ turn number} \quad A_z = |A_z|\mathrm{e}^{i\delta_z} \equiv \text{ constant of motion}$$

$$\Phi_z \equiv \mu_z - Q_z\theta \quad \text{(periodic phase function)}$$

Twiss functions:

$$\beta_z^j = |Z_j(Q_z)|^2/A_z^2 \qquad \mu_z^j = \arg\,(Z_j) - \delta_z$$

$$Z_j(Q_z) \equiv \text{ Fourier component of } z_j$$

Amplitude fit:

$$|A_z|^2 = \frac{\sum_j 1/\beta_z^j}{\sum_j 1/|Z_j(Q_z))|^2} \simeq \frac{\sum_j 1/\beta_z^{0j}}{\sum_j 1/|Z_j(Q_z))|^2}$$

Using Mais-Ripken parameterization, motion in presence of coupling may be written as

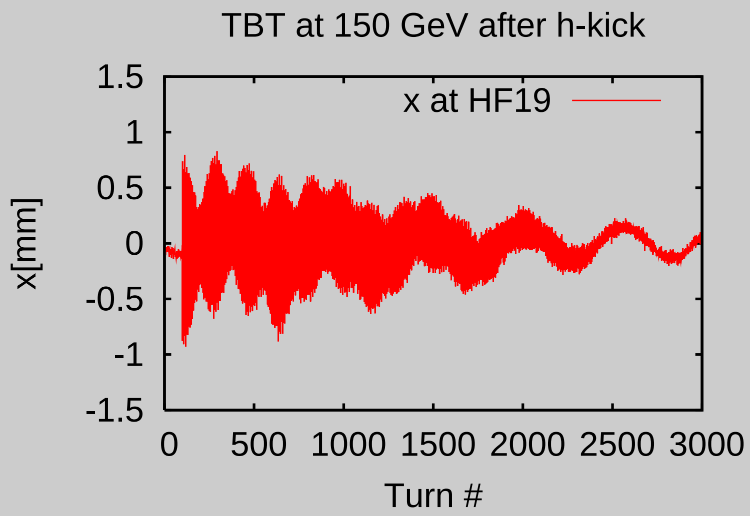
$$
\begin{aligned}
x_n = {} & A_I\sqrt{\beta_{xI}}\cos(\phi_{xI}+\delta_I+2\pi nQ_I)+\\
& A_{II}\sqrt{\beta_{xII}}\cos(\phi_{xII}+\delta_{II}+2\pi nQ_{II})\\
y_n = {} & A_I\sqrt{\beta_{yI}}\cos(\phi_{yI}+\delta_I+2\pi nQ_I)+\\
& A_{II}\sqrt{\beta_{yII}}\cos(\phi_{yII}+\delta_{II}+2\pi nQ_{II})
\end{aligned}
$$

and thus the Fourier analysis gives the coupled Mais-Ripken Twiss functions $\beta_{zI,II}$ and $\phi_{zI,II}$ $(z \equiv x, y)$, a part for the constants of motion $A_{I,II}$ and $\delta_{I,II}$.

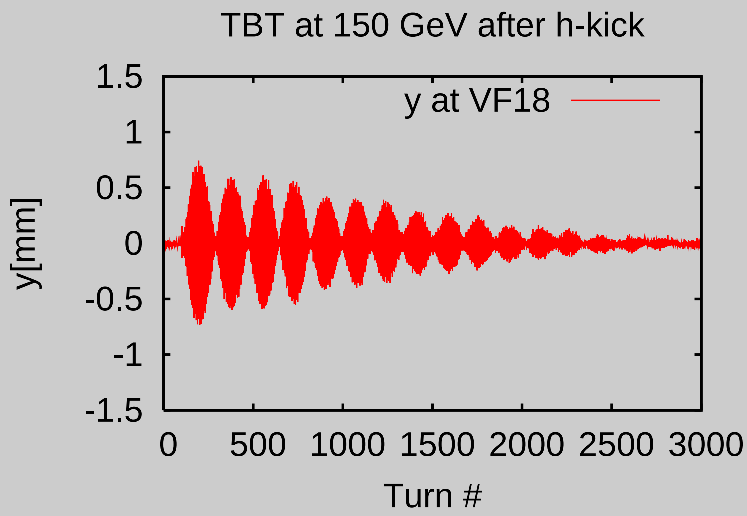
Some Tevatron results

The Tevatron BPMs can store 8192 positions data per BPM. The electronics upgrade allows a high resolution ($\simeq$ 15- 50 μm) measurement of the TBT beam position. Under "ideal" conditions the oscillations following a kick last some thousand turns

TBT position after a horizontal kick

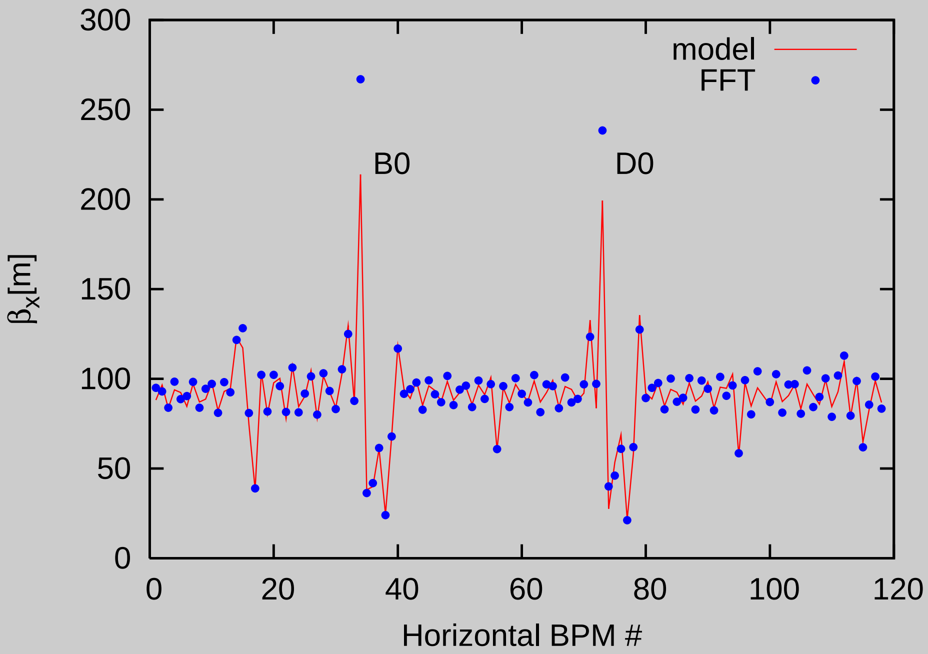


TBT position at HF19

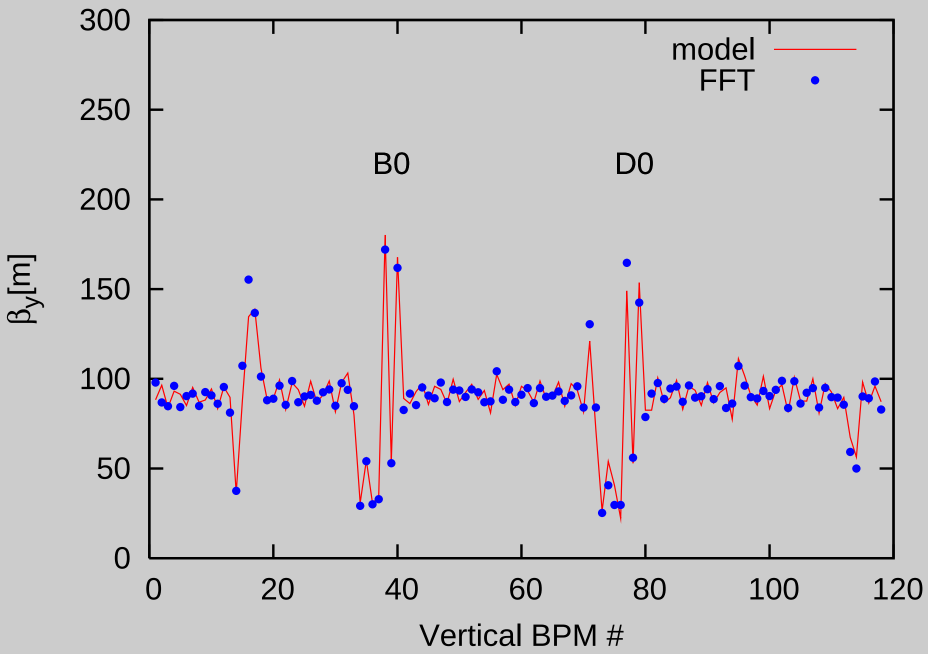


TBT position at VF18

Reconstructed Injection Optics (November 2005 data)



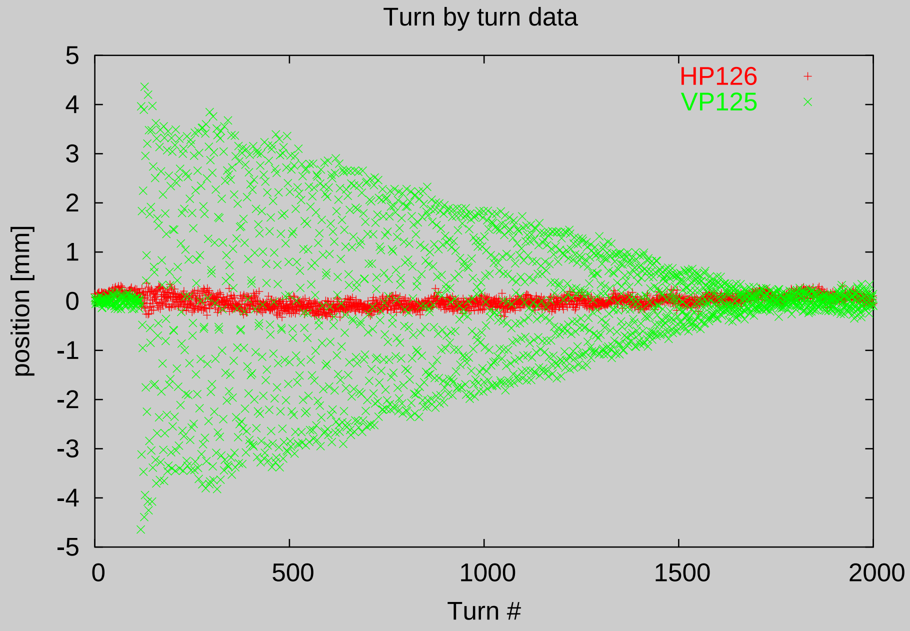
Horizontal

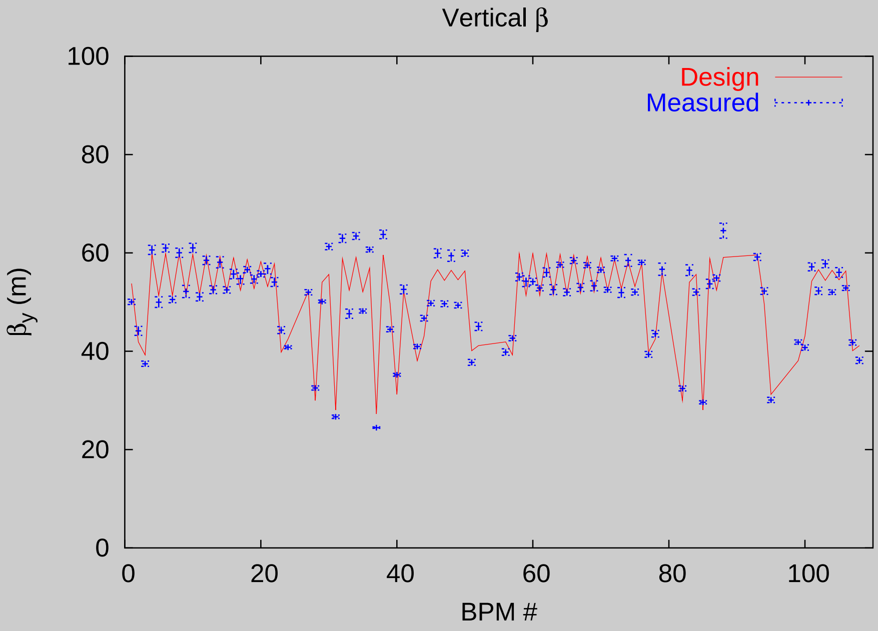


Vertical

Some Main Injector results (2007)

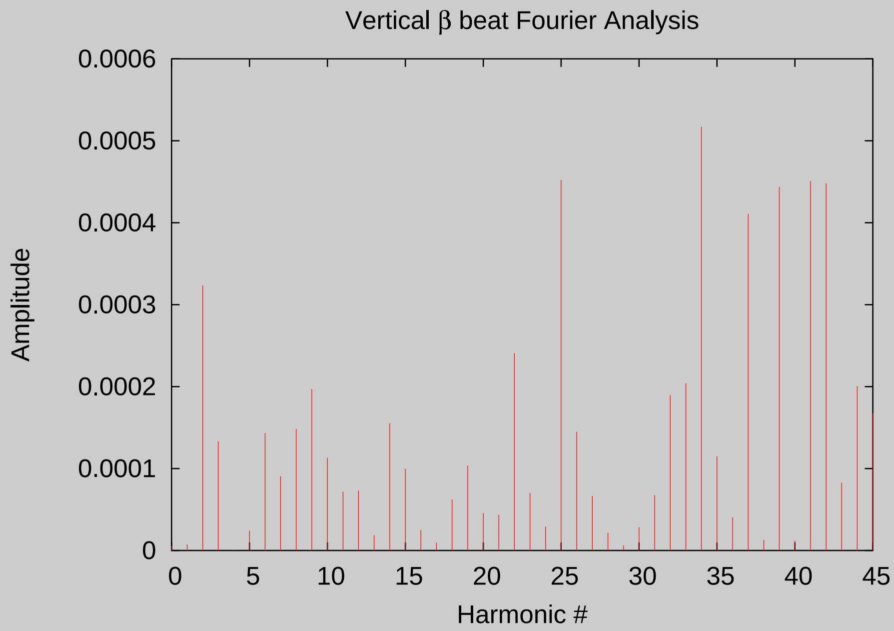
The upgrade of the MI BPM system allows the use of TBT techniques.

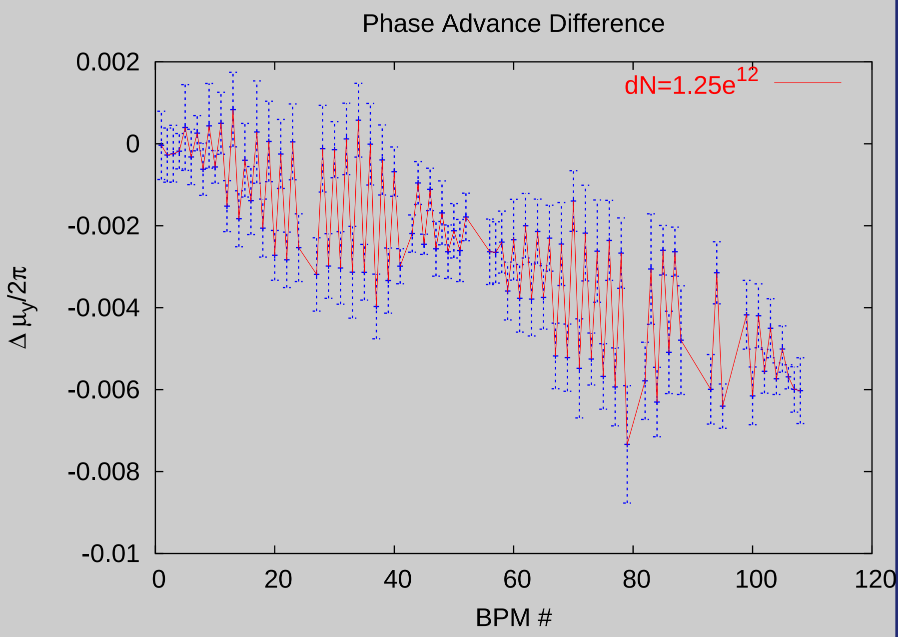




TBT following a v-kick
no coupling into horizontal plane

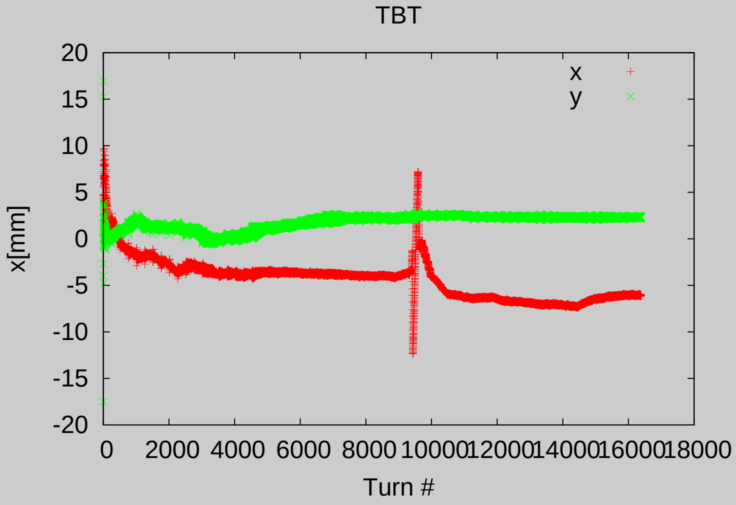
Vertical β vs. MAD model



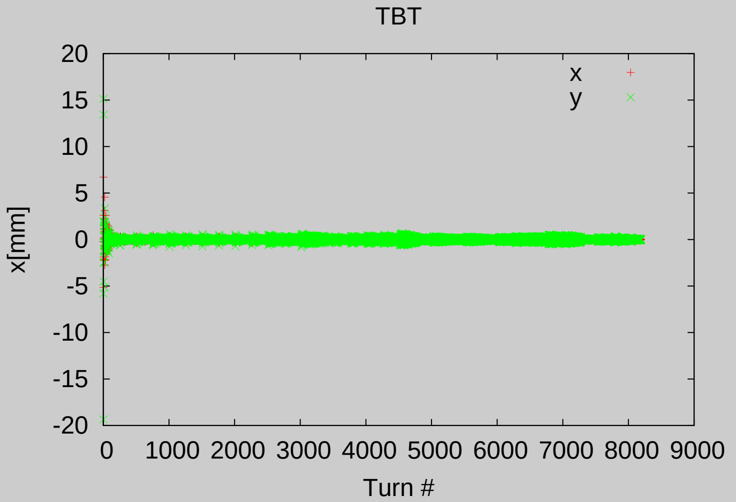


Fourier analysis of $\Delta\beta_z(s)$
does not show a peak where expected (42)
but a piecewise fit gives reasonable results

$\Delta\mu_z$ for two beam currents
No evidence of a localised
impedance source



TBT raw data

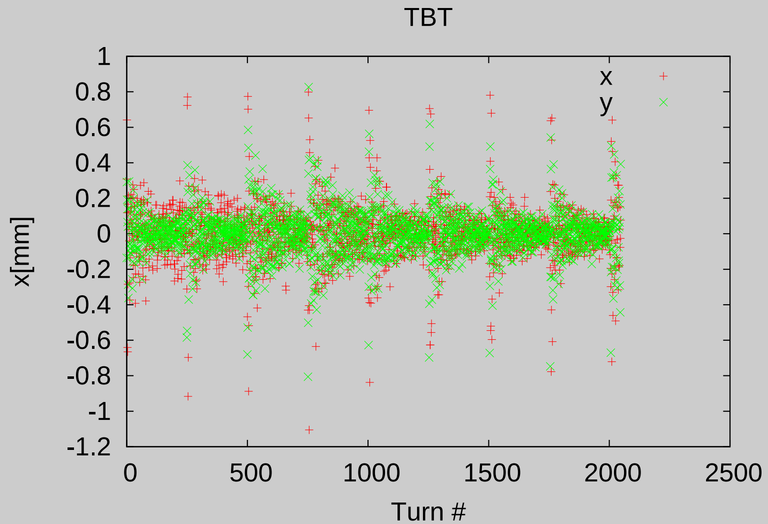


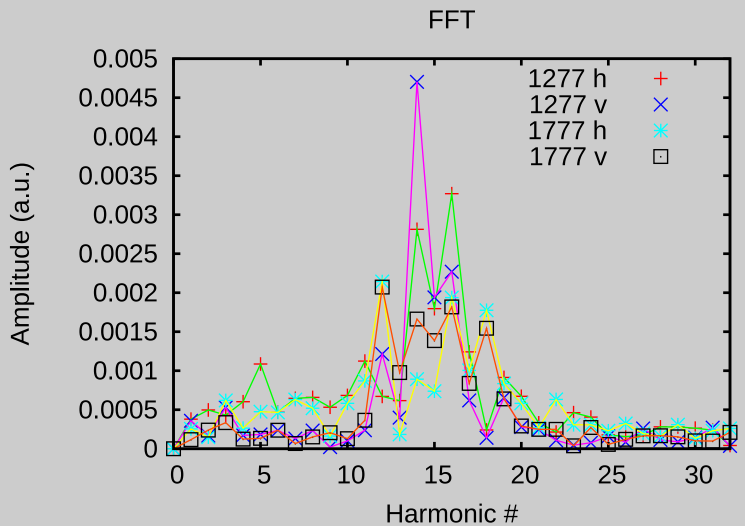
after "smoothing"

Some Booster results (2008)

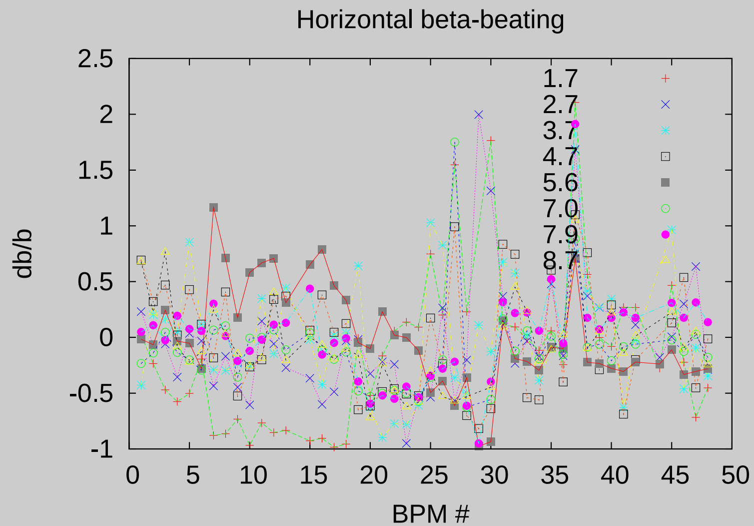
The Booster accelerates the proton beam from E_k=400 MeV to 8 GeV in 33 ms. There are 51 BPMs measuring in both planes, each recording about 20000 turns. The orbit has a large excursion on the ramp; the closed orbit is piecewise computed and subtracted from the raw data.

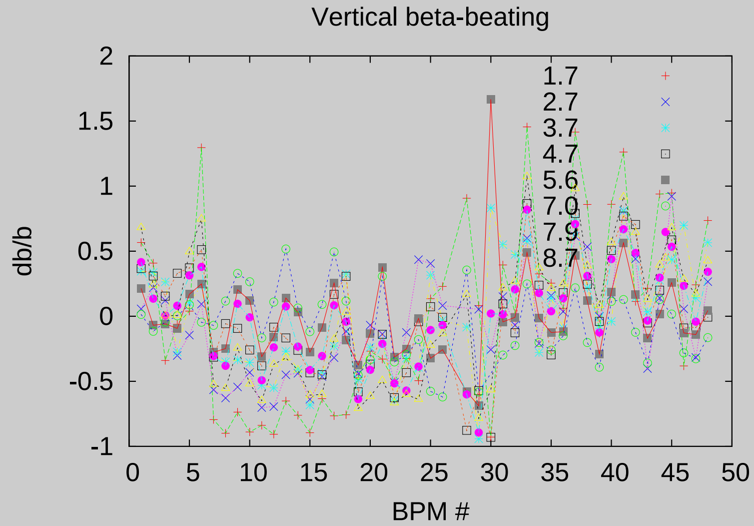
The beam is kicked every 500 turns either by a horizontal or a vertical pinger. Spectra may be confusing.

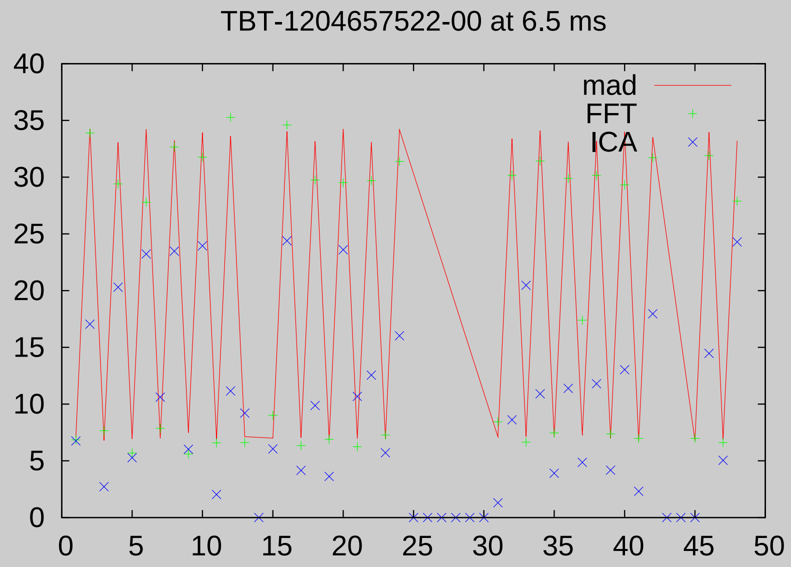




The β-beat wrt MAD model is very large, it must be an artifact

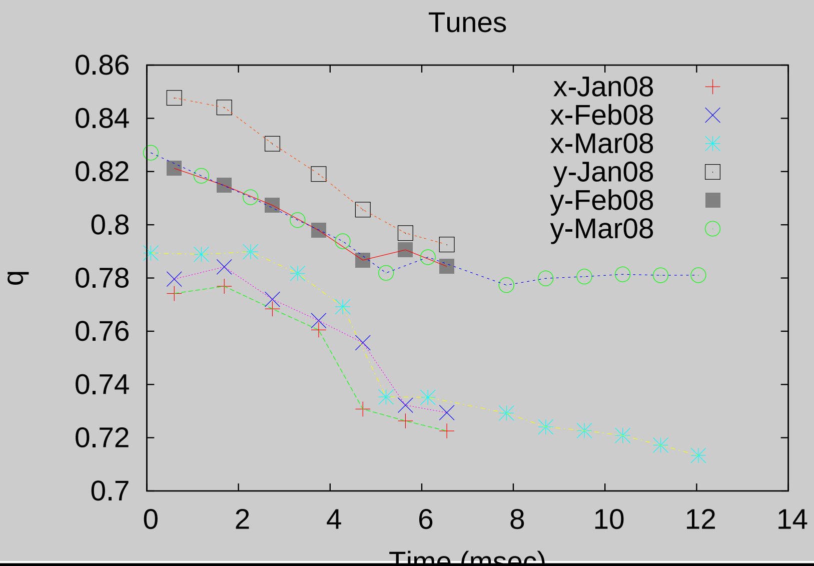






Some data are good enough for a sensible optics computation; for instance for the 6.5 ms slot of a ramp recorded in March 2008.

nb: the points denoted as ICA have been obtained with my "home made" version.



At least it is possible to track the tunes; this allowed to correct the linear coupling in the first 4 ms of a test ramp. Efforts to move tunes for large space charge operation are going on.

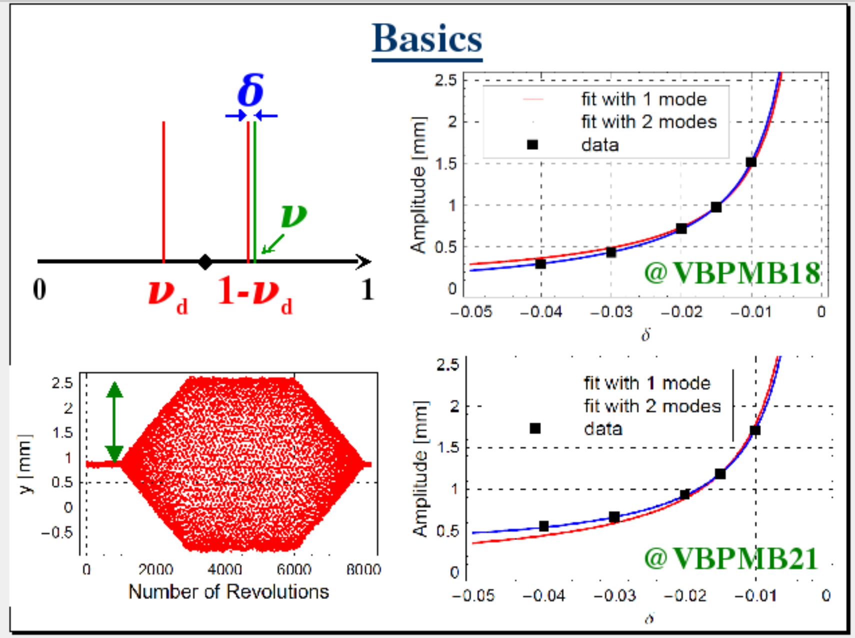
AC dipole

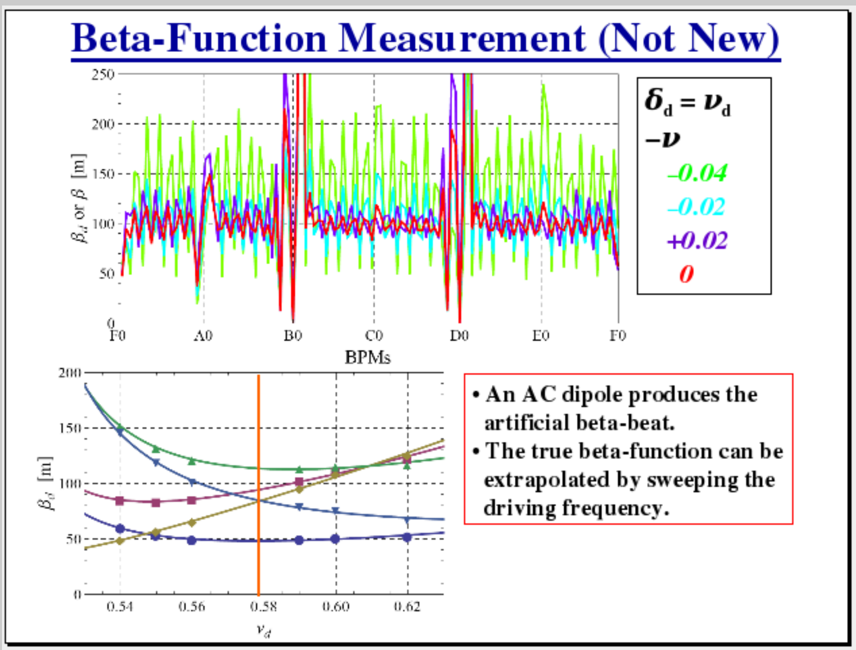
The coherent oscillation following a single kick *decays* more or less quickly and the *emittance growth* makes the beam almost unusable afterwards.
One can use a AC dipole [a] for exciting a driven coherent oscillation. Although the dipole frequency, ν_d, is very close to the natural beam oscillation frequency, ν, if adiabatically ramped up and down and if the field is small enough, it does not blow-up the emittance.

AC dipoles have been employed at BNL (AGS and RHIC), CERN SPS and at Tevatron. There is an ongoing project to develop AC dipoles for LHC too.

The relationship between TBT analysis results and actual BPMs Twiss parameters is not as straightforward as for free oscillations: the AC dipole is *equivalent* to a *quadrupole perturbation* [b] which vanishes only when $\delta\nu = \nu - \nu_d$ vanishes. For hadron machines this condition cannot be fullfilled, but one can make several measurements for different values of $\delta\nu$ and fit the results to find the *unperturbed* Twiss parameters at the BPMs location.

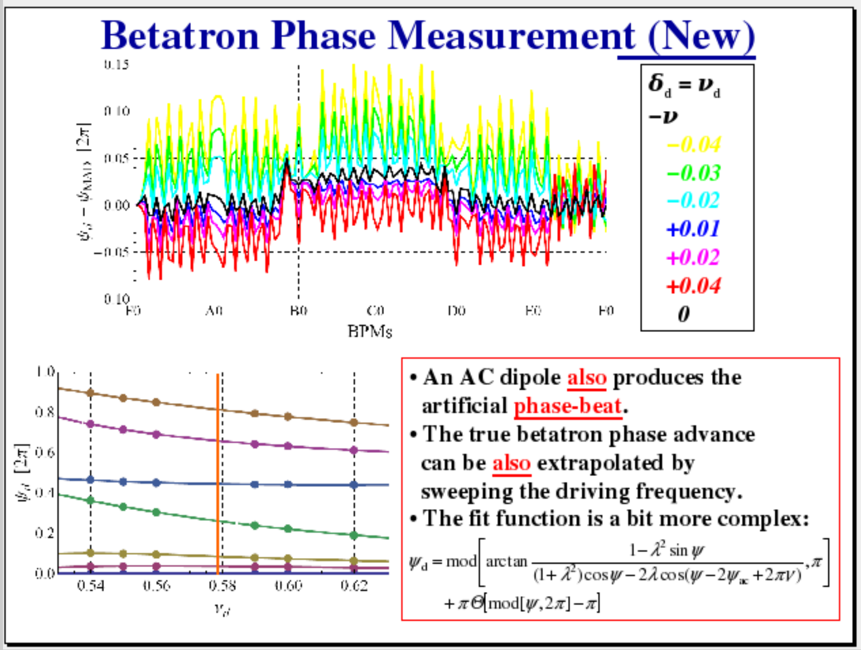
[a] sinusoidally oscillating magnetic field
[b] see Miyamoto, Kopp, Jansson and Syphers, THPAN102, PAC07





Tevatron AC dipole

(Ryoichi Miyamoto courtesy)



Independent Component Analysis of TBT data (ICA)

ICA uses techniques for *blind source separation*. These techniques are used in signal processing for recovering a set of signals of which only instantaneous linear mixtures are observed by exploiting their time coherence. It is assumed that the sources are *narrowband* and *independent*. The matrix containing P measurements at M stations, X is written as

$$X = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1P} \\ x_{21} & x_{21} & \dots & x_{2P} \\ \dots & \dots & \dots & \dots \\ x_{M1} & x_{P2} & \dots & x_{MP} \end{pmatrix} = AS + \mathcal{N}$$

$M \equiv$ number of BPMs

$P \equiv$ number of turns

$N \equiv$ number of sources

where S is a $N \times P$ matrix describing the N sources, A is a $M \times N$ *mixing matrix* and $\mathcal{N}$ is a $M \times P$ matrix containing the measurement noise.

One can make a SVD decomposition of the starting covariance matrix $\boldsymbol{C^x(0)}$ in order to keep only the $\boldsymbol{N_s}$ singular values above a given threshold [a]

$$\boldsymbol{C^x(0)} = \begin{pmatrix} \boldsymbol{U_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{U_2} \end{pmatrix} \begin{pmatrix} \boldsymbol{\Sigma_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Sigma_2} \end{pmatrix} \begin{pmatrix} \boldsymbol{U_1^T} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{U_2^T} \end{pmatrix}$$

$\boldsymbol{U_{1,M\times N_s}}$ and $\boldsymbol{\Sigma_{1,N_s\times N_s}}$ collect the parts corresponding to the singular values *above* threshold.

Thus the matrices

$$\boldsymbol{V_{N_s\times M} \equiv \Sigma_1^{-1/2} U_1^T} \quad \text{and} \quad \boldsymbol{\xi_{N_s\times P} \equiv V X}$$

are constructed.

Due to the hypothesis of *narrowband* and *independence* and in the limit $\boldsymbol{P \to \infty}$, for the *time lagged* covariance matrix of $\boldsymbol{S}$, $\boldsymbol{C^s_{ij}(n)}$, holds

$$\boldsymbol{C^s_{ij}(n) \equiv (SS^T)_{ij}(n) = \Sigma_k S_{ik} S_{j,k+n} = \delta_{ij} S_{ij}(n) \Rightarrow C^s(n)} \text{ is } \textit{diagonal}$$

[a] the SVD of a symmetric matrix is equivalent to the eigenvalue problem

Thus the time lagged covariance matrices $C^x(n)$ and $C^\xi(n)$ are symmetric. In particular

$$C^\xi(n) = [\Sigma_1^{-1/2} U_1^T A] \, C^s(n) \, [\Sigma_1^{-1/2} U_1^T A]^T \equiv W C^s(n) W^T$$

We recognize that

- this a "similarity transformation"
- for each n the *same* transformation W maps the starting basis into the basis of eigenvectors of $C^\xi(n)$

Now the game consists in finding the transformation which diagonalizes *simultaneously* all $C^\xi(n)$, with $n \in [n_1, n_m]$ [a]. There are mathematical algorithms to find an (approximated) solution.
It is worth noting that having applied the ICA to $C^\xi(n)$ rather than $C^x(n)$ have greatly reduced the dimensions of the problem.

[a] the previous analysis may be applied to the, *by definition*, symmetric matrix

$$\bar{C}^\xi(n) \equiv \frac{1}{2} \left[C^\xi(n) + C^{\xi T}(n) \right]$$

rather than $C^\xi(n)$

If $\boldsymbol{W}$ is the $\boldsymbol{N_s} \times \boldsymbol{N_s}$ matrix which diagonalizes the given set of $\boldsymbol{C^{\xi}(n)}$ the (important part of) mixing matrix $\boldsymbol{A}$ and source matrix $\boldsymbol{S}$ are given by

$$\boldsymbol{A = V^{-1}W \qquad S = W^T V X}$$

Betatron motion:

$$\begin{Bmatrix} S_{ik} \\ S_{i+1,k} \end{Bmatrix} = \begin{Bmatrix} \cos 2\pi Q_i(k-1) \\ \sin 2\pi Q_i(k-1) \end{Bmatrix}$$

$$\begin{Bmatrix} A_{ji} \\ A_{j,i+1} \end{Bmatrix} = \begin{Bmatrix} a_i\sqrt{\beta_{ji}}\sin(\mu_{ji} + \Phi_i) \\ a_i\sqrt{\beta_{ji}}\cos(\mu_{ji} + \Phi_i) \end{Bmatrix}$$

The Twiss functions are finally given by

$$\beta_i = a^2(A^2_{ni} + A^2_{\bar{n}i}) \qquad \psi_i = \phi_o + \tan^{-1}\left(\frac{A^2_{ni}}{A^2_{\bar{n}i}}\right)$$

where $\boldsymbol{a}$ and $\boldsymbol{\phi_0}$ are constant of motion and $\boldsymbol{n}$ and $\boldsymbol{\bar{n}}$ are the indeces corresponding to the betatron motion component. They are recognized by a Fourier analysis of the temporal vectors of $\boldsymbol{S}$ (its rows) or of the reconstructed contribution of each source to $\boldsymbol{X_{ij}}$ $(\boldsymbol{i = 1, ...M}$ and $\boldsymbol{j = 1, ...P})$.

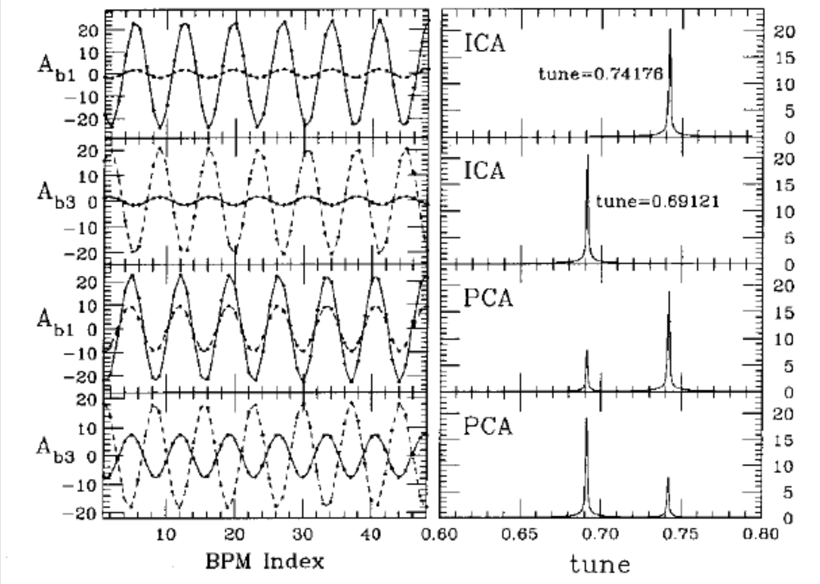
- ICA may be more convenient than the simple Fourier analysis in presence “spurious” sources as synchrotron side-bands
- an evaluation of the constant $\boldsymbol{a}$ is needed for which a *model* is needed
- the different modes are recognised by a Fourier analysis
- wrong BPMs, recognised by analysing the spatial vectors of $\boldsymbol{A}$, must be excluded and the analysis repeated

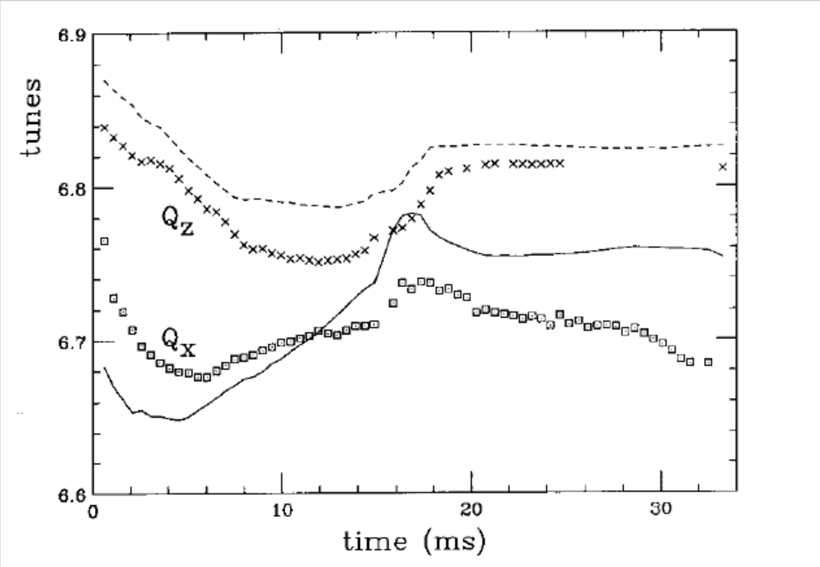
Different ICA algorithms have been implemented for Booster and Tevatron

- a MATLAB application for the Booster by X. Huang is available on wally
- a OCTAVE version for TEVATRON by A. Petrenko runs on HEIMDALL (see http://www-bdnew.fnal.gov/tevatron/lifetrac/tbt/)

But we have *no* keepers....

Some results for Booster[a]

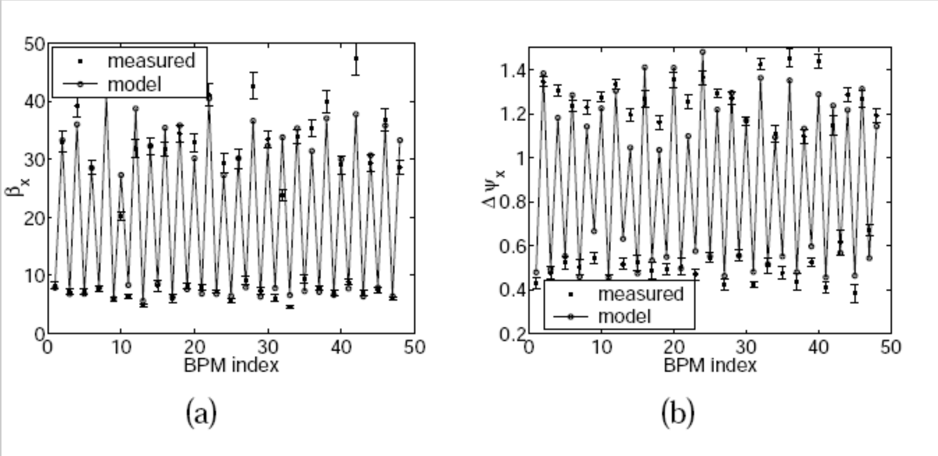




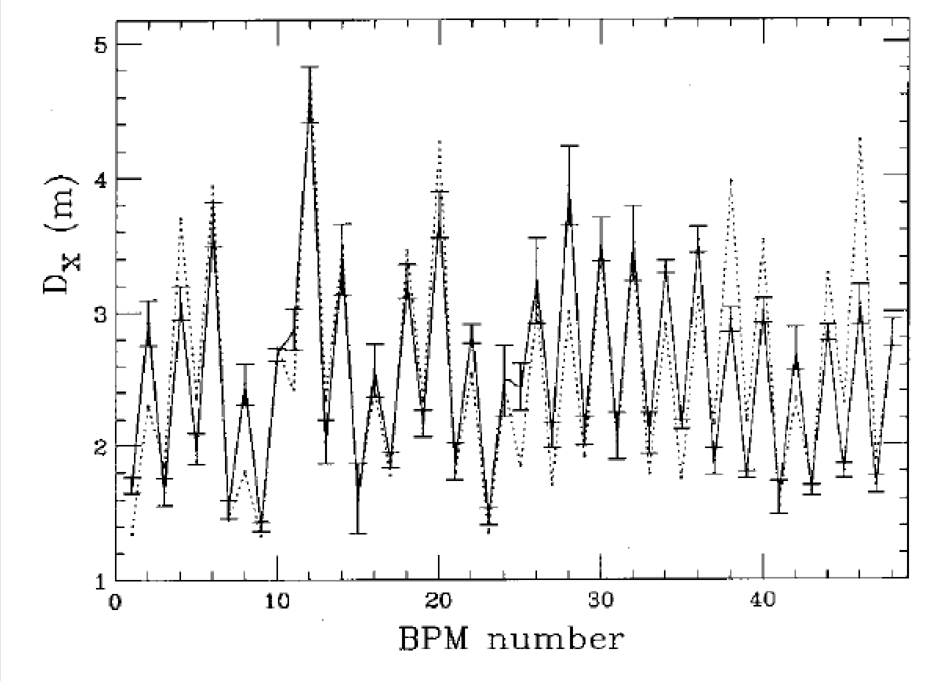
ICA compared to Principal Component Analysis for *simulated* data in presence of coupling: the spectrum of the temporal modes after ICA decomposition shows no contamination between the two orthogonal modes.

Tunes tracked along the Booster ramp (measurement)

[a] from Huang, Lee, Prebys and Tomlin,Phys.Rev.ST Accel.Beams 8:064001(2005) and PAC05

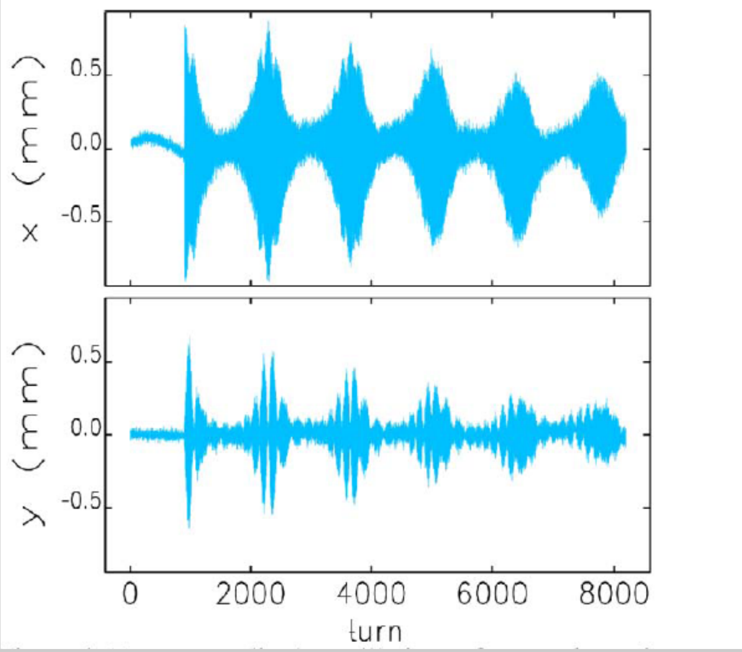


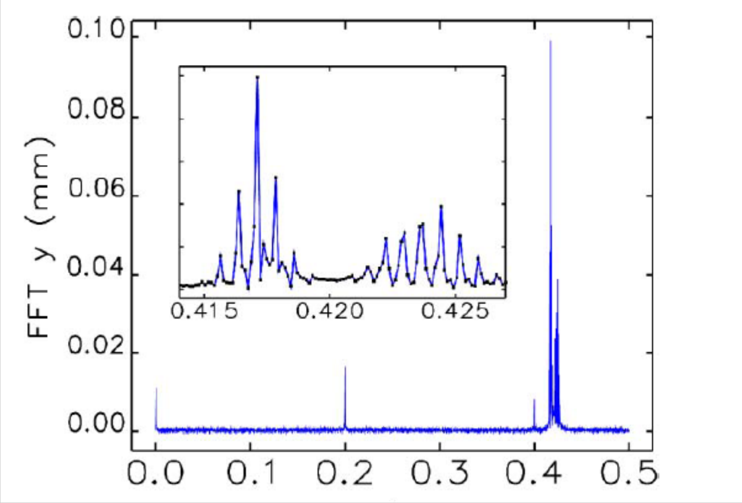
Twiss functions from TBT measurement in DC mode vs. MAD model



Dispersion from TBT measurement in DC mode vs. MAD model

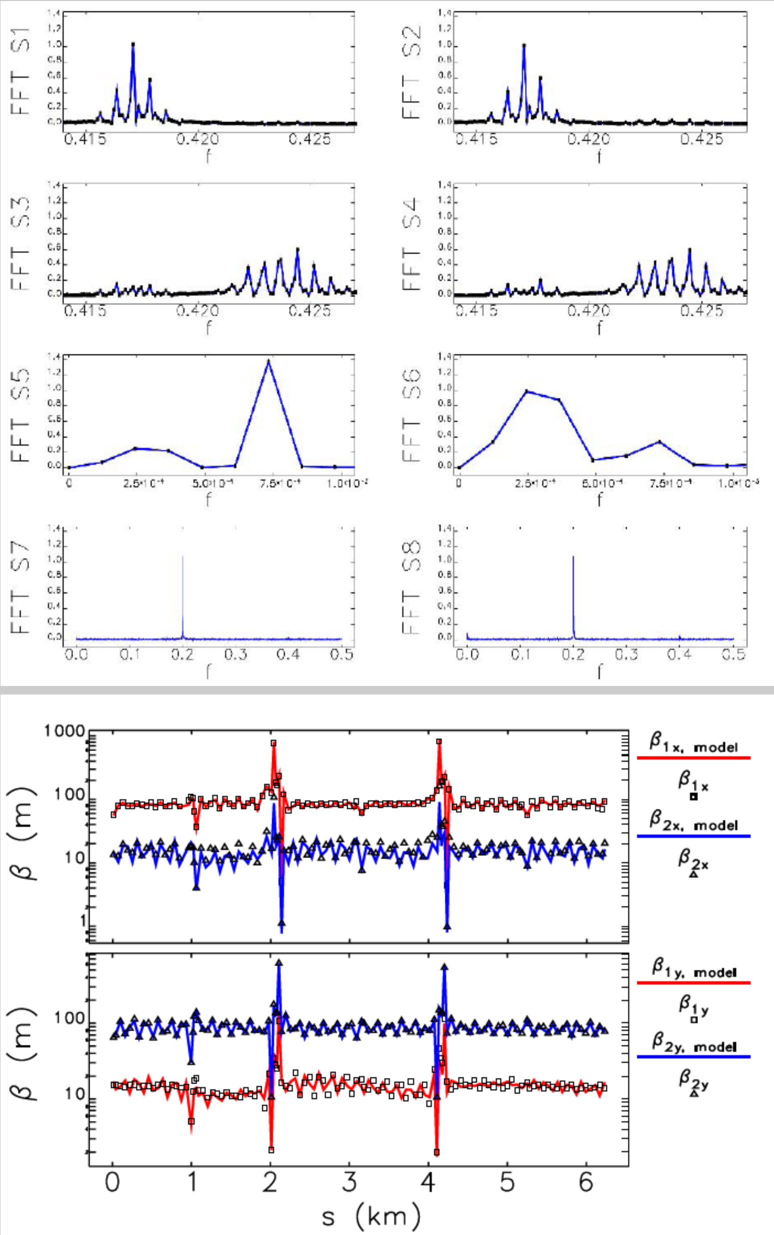
Some results for Tevatron[a]





TBT position after a horizontal kick, large coupling visible in the TBT data and its spectrum

[a] from Petrenko, Lebedev and Valishev, WEPP037, Epac08



The spectrum of the temporal modes after ICA decomposition, ie rows or of the reconstructed $\boldsymbol{S}$, shows small contamination between orthogonal modes.

OPTICS FIT FROM TBT DATA

The Fourier analysis, for instance[a], of the measured TBT data

$$\begin{aligned} x_n = & A_I\sqrt{\beta_{xI}}\cos(\phi_{xI} + \delta_I + 2\pi n Q_I) + \\ & A_{II}\sqrt{\beta_{xII}}\cos(\phi_{xII} + \delta_{II} + 2\pi n Q_{II}) \\ y_n = & A_I\sqrt{\beta_{yI}}\cos(\phi_{yI} + \delta_I + 2\pi n Q_I) + \\ & A_{II}\sqrt{\beta_{yII}}\cos(\phi_{yII} + \delta_{II} + 2\pi n Q_{II}) \end{aligned}$$

gives the coupled Mais-Ripken twiss functions $\beta_{zI,II}$ and $\phi_{zI,II}$ $(z \equiv x, y)$, a part for the constants of motion $A_{I,II}$ and $\delta_{I,II}$.

[a] Other TBT data analysis methods of course may be used

The eigenvectors of the coupled transport matrix are related to the Mais-Ripken twiss functions

$$
\begin{aligned}
V_{11} &\equiv \sqrt{\beta}_{xI} \cos \phi_{xI} & V_{12} &\equiv \sqrt{\beta}_{xI} \sin \phi_{xI} \\
V_{13} &\equiv \sqrt{\beta}_{xII} \cos \phi_{xII} & V_{14} &\equiv \sqrt{\beta}_{xII} \sin \phi_{xII} \\
V_{31} &\equiv \sqrt{\beta}_{yI} \cos \phi_{yI} & V_{32} &\equiv \sqrt{\beta}_{yI} \sin \phi_{yI} \\
V_{33} &\equiv \sqrt{\beta}_{yII} \cos \phi_{yII} & V_{34} &\equiv \sqrt{\beta}_{yII} \sin \phi_{yII}
\end{aligned}
$$

Goal: adjust

- quadrupole gradient and tilt
- BPMs calibration and tilt
- $A_{I,II}$ and $\delta_{I,II}$

in order to fit the measured eigenvector values at the BPMs.

Application to Tevatron

- Number of observation points: 2×118
- Current Tevatron model (A.Valishev): 216 normal and 216 skew thin quadrupoles to simulate gradient and tilt errors. We must add the unknown BPM calibrations and tilts, with the additional condition $< r_i >=1$, and the oscillation amplitude and phase.

All together: 908 parameters and 945 constraints. As for LOCO we need a code for computing the model optics. After trying with MADX, a dedicated program for both optics computations and minimization has been written[a]. To save computing time machine sections between variable elements and/or between observation points are described by pre-computed maps.

The advantage wrt for instance LOCO is the rapidity of the data acquisition.

The f77 code has been translated into C by using f2c. The code has been embedded in the C++ console application W116 for acquiring data and setting fit parameters.

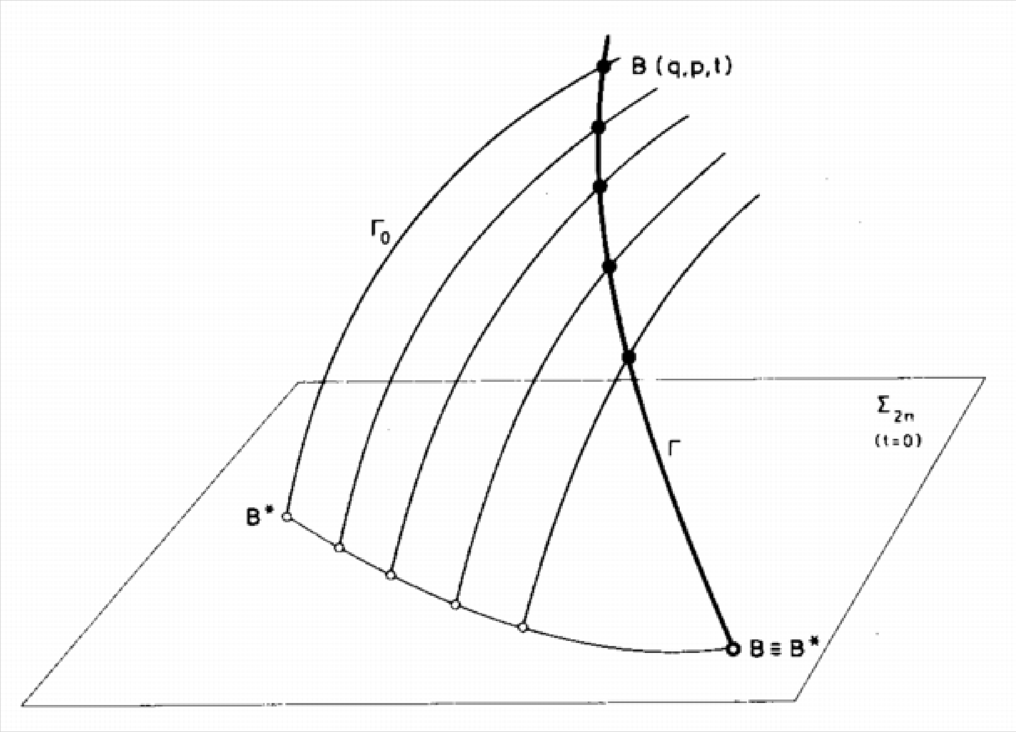
---

[a] by guest scientist V.Kapin, based on Y.Alexahin Mathematica Notebook

Linear Coupling (perturbation theory)

Method of the variation of constants:

The general solution of the perturbed motion keeps the form of the unperturbed one with constants, c_i, depending on time[a]



(Guignard, CERN 78-11)

Hamiltonian in presence of a perturbation H_1

$$H = [H_0 + H_1](q_1, ...q_n, p_1, ...p_n)$$

$$= [U_0 + U_1](c_1, ...c_{2n})$$

Equations of motion

$$\frac{dc_j}{dt} = \Sigma_m [c_j, c_m] \frac{\partial U_1}{\partial c_m}$$

[a] θ or s in our case

When the unperturbed Hamiltonian describe the betatron motion, thus

$$\frac{dA_z}{d\theta} = i\frac{\partial U_1}{\partial A_z^*} \qquad \frac{dA_z^*}{d\theta} = -i\frac{\partial U_1}{\partial A_z} \quad \text{with } q_i = \frac{A_z}{2}\sqrt{\beta_z}e^{i\mu_z} + c.c.$$

For perturbation fields generating linear coupling (Guignard)

$$U_1(\vec{a}) = \frac{1}{2}[C_+(\theta)a_x a_y + C_+^*(\theta)a_x^* a_y^* + C_- a_x a_y^* + C_-^* a_x^* a_y]$$

$$a_z \equiv A_z e^{iQ_z\theta}$$

where

$$C_\pm(\theta) \equiv \frac{R\sqrt{\beta_x\beta_y}}{2B\rho}\left\{\left(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\right) + B_\theta\left[\left(\frac{\alpha_x}{\beta_x} - \frac{\alpha_y}{\beta_y}\right) - i\left(\frac{1}{\beta_x} \mp \frac{1}{\beta_y}\right)\right]\right\} \mathrm{e}^{i(\Phi_x \pm \Phi_y)}$$

and

$$\Phi_z \equiv \mu_z - Q_z\theta$$

"Ansatz" (Y. Alexahin)

$$a_x(\theta) = a_{x0}(\theta) + w_-^*(\theta) a_{y0}(\theta) + w_+^*(\theta) a_{y0}^*(\theta)$$

$$a_y(\theta) = a_{y0}(\theta) - w_-(\theta) a_{x0}^*(\theta) + w_+^*(\theta) a_{x0}^*(\theta)$$

Inserting into the equation of motion and keeping 1$^{\text{th}}$ order terms one finds the equations for $w_\pm$

$$2i\mathrm{e}^{-iQ_\pm\theta} \frac{d}{d\theta} \mathrm{e}^{iQ_\pm\theta} w_\pm(\theta) = C_\pm(\theta)$$

The periodic solutions are

$$w_\pm(\theta) = -\int_0^{2\pi} d\theta' \frac{C_\pm(\theta')}{4\sin\pi Q_\pm} \mathrm{e}^{-iQ_\pm[\theta-\theta'-\pi \mathrm{sign}(\theta-\theta')]}$$

with

$$Q_\pm \equiv Q_x \pm Q_y$$

The functions $\tilde{w}_\pm \equiv w_\pm \mathrm{e}^{iQ_\pm\theta}$ are

- constant in coupler free regions
- experience a discontinuity $-iC_\pm\ell/2R$ at coupler locations $\Rightarrow$ diagnostics tool !
- are constant on the resonances $Q_x \pm Q_y = int$.

Minimum tune split (Guignard)

$$\Delta \equiv |\bar{C}_-^{n_-}| \qquad \bar{C}_\pm^{n_\pm} = \frac{1}{2\pi}\int_0^{2\pi} d\theta\, C_\pm \mathrm{e}^{in_\pm\theta} = \frac{n_\pm - Q_\pm}{\pi}\int_0^{2\pi} d\theta\, w_\pm \mathrm{e}^{in_\pm\theta}$$

with

$$n_\pm \equiv \mathrm{Round}(Q_x \pm Q_y)$$

Linear coupling computation through TBT analysis

TBT beam position at the j[th] vertical BPM following a horizontal kick

$$y_n^j = \left[\sqrt{\beta_y^j}\left(\mathrm{e}^{-i\Phi_y^j} w_+^j - \mathrm{e}^{i\Phi_y^j} w_-^j\right)\right] A_x \mathrm{e}^{iQ_x(\theta_j + 2\pi n)} + c.c.$$

TBT beam position at the j-th horizontal BPM following a vertical kick

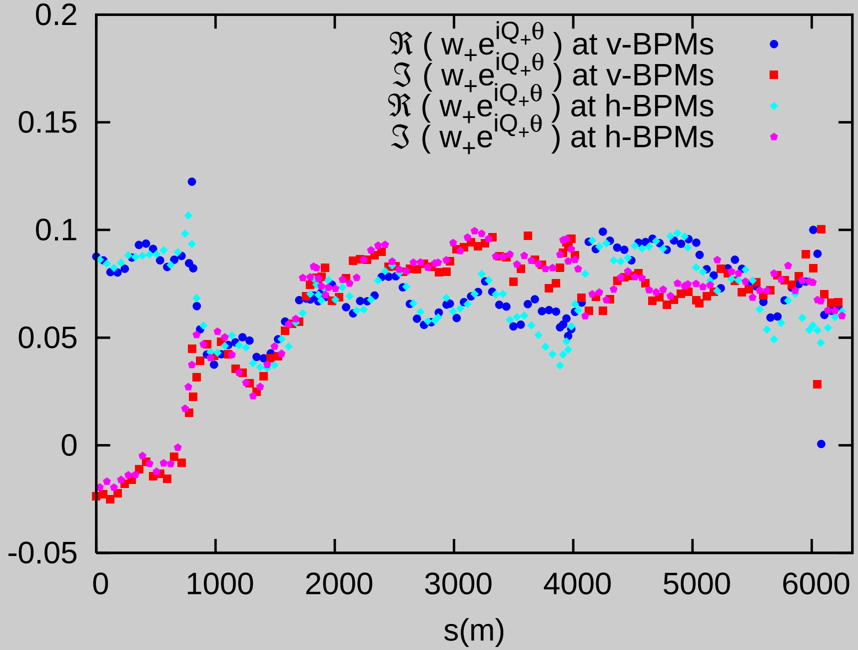
$$x_n^j = \left[\sqrt{\beta_x^j}\left(\mathrm{e}^{-i\Phi_x^j} w_+^j + \mathrm{e}^{i\Phi_x^j} w_-^{*j}\right)\right] A_y \mathrm{e}^{iQ_y(\theta_j + 2\pi n)} + c.c.$$

The FFT of y^j at Q_x, $Y^j(Q_x)$, for a horizontal kick ($X^j(Q_y)$ for a vertical one) is proportional to the *coupling functions* $w_\pm(\theta_j)$.

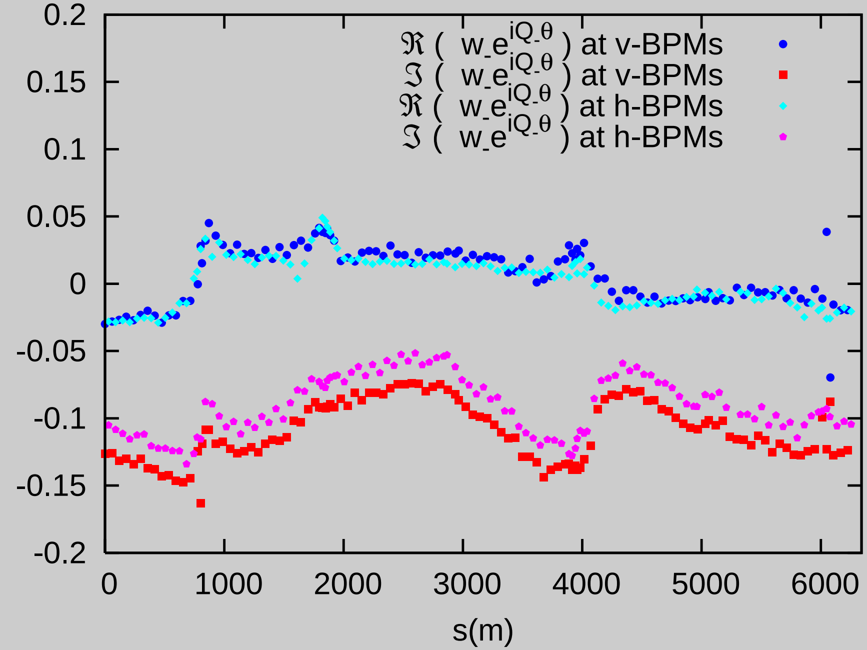
We get per each BPM 2 real equations in 4 unknowns. When between two consecutive monitors there are no strong source of coupling, the four equations can be solved in favor of $w_\pm(\theta_j) = w_\pm(\theta_{j+1})$.

Examples of Tevatron Measurements

Coupling functions (November 2005 data)



$\tilde{w}^{+}$



$\tilde{w}^{-}$

Jumps visible around 1000 (SQA0), 1500 (A38) and 4000 (D16) meters.

The TBT analysis may be used to correct the linear coupling.

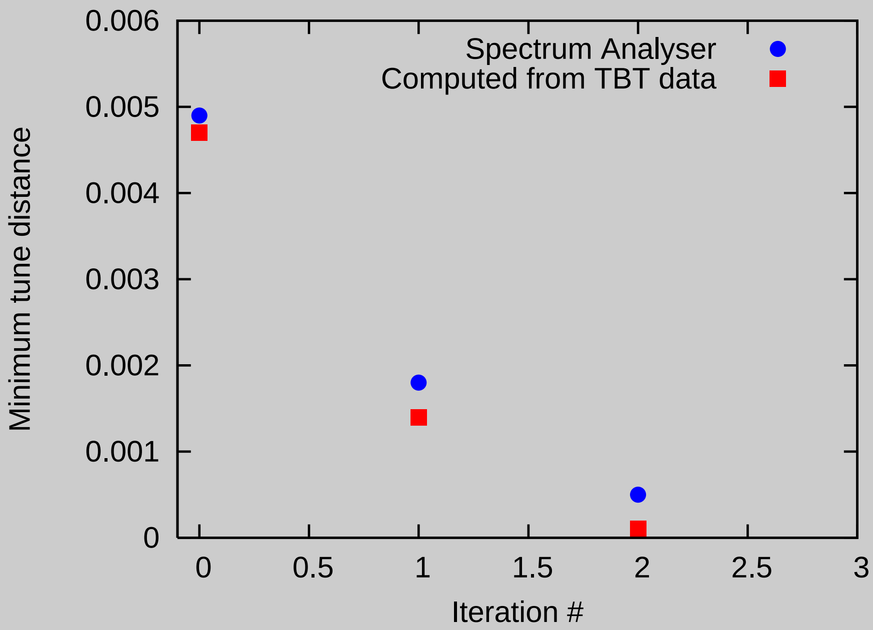
An application program for the TBT analysis has been integrated in the TEVATRON control system and is used routinely at shot set up for correcting the minimum tune split $\Delta \equiv |\bar{C}_-|$ with two skew quadrupole circuits.

The application

- fires the pinger
- acquires the TBT data
- perform the Fourier analysis
- computes the coupling functions, $w^{\pm}$ at the BPMs and the integral $\bar{C}_-^{n-}$
- sets the compensating currents in the skew quadrupoles circuits

This procedure is *faster* than finding empirically the minimum tune split.

Minimum tune split measured with S.A. and computed from TBT data

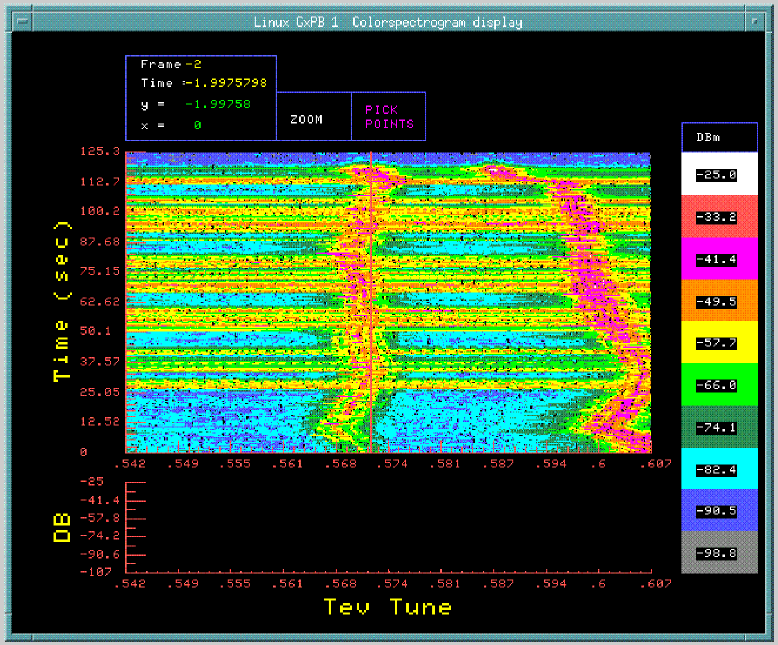


● Spectrum Analyser

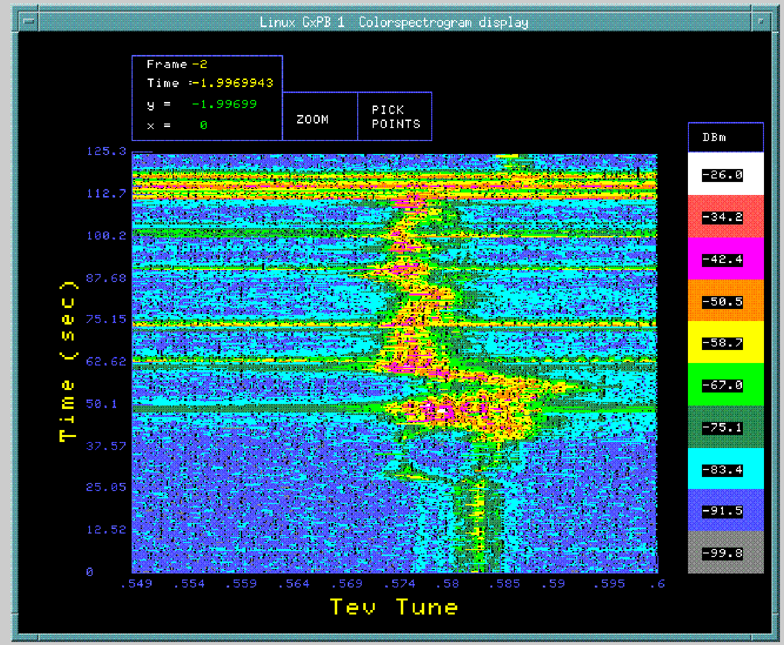
■ TBT Analysis

TEVATRON being a fast ramping machine (83 seconds from 150 to 980 GeV), the TBT analysis is a very practical method for measuring optics and coupling also during acceleration. A second console application (W118) has been written for this purpose.

First ramp after 2006 shut down (3th June 2006)



After correcting with W118 (6th June 2006)



Summary

I have tried to highlight some of the optics tools developed for Run II, mainly for Tevatron.

Some of them found application also in other machines.

Lack of time (and knowledge) did not allow me to show other tools developed by various colleagues.